Exercise XI, Theory of Computation 2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long.

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

More Problems on Complexity Classes

1 Define **coNP** $= \{L : \overline{L} \in \mathbf{NP}\}$ as the class of languages whose complements are in **NP**. Show that if any **NP**-complete problem lies in **coNP**, then **NP** = **coNP**.

2 Denote by **RE** the class of recognisable languages. We say that a language L is **RE**-*complete* iff $L \in \mathbf{RE}$ and for every $L' \in \mathbf{RE}$ we have $L' \leq_m L$. Show that HALT is **RE**-complete.

Problems on Circuit Complexity

3 Complete the proof of Circuit-SAT $\leq_p$ SAT from the lecture by finding an equivalent CNF formula for each of the three logical predicates

$$y \leftrightarrow (x \vee z), \quad y \leftrightarrow (x \wedge z) \quad \text{and} \quad y \leftrightarrow \neg x.$$

4 The function $\text{XOR}_n \colon \{0,1\}^n \to \{0,1\}$ outputs 1 iff the number of 1-bits in the input is odd. Show that XOR_n can be computed with a boolean circuit (gates $\vee$, $\wedge$, $\neg$) of size $O(n)$.

Hint: *Construct a circuit for $n = 2$ and then use many copies of that circuit for general n.*

5 Let φ be any DNF formula over n variables that computes XOR_n. Recall that $\varphi = T_1 \vee \cdots \vee T_m$ where each T_j is a *term*, that is, a conjunction of literals.

5a* Show that any term T_j either contains n distinct variables or is *contradictory*, meaning that it contains x_i and $\overline{x}_i$ for some variable x_i.

Hint: *Use the fact that the value of XOR_n is flipped if we flip the value of any x_i.*

5b Thus, show that φ contains at least 2^{n-1} terms.

Note that problems 3–4 together imply that circuits can be much more expressive than CNFs.